Strings from Membranes and Fivebranes

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Under the six-dimensional heterotic/type IIA duality map, a solitonic membrane solution of heterotic string theory transforms into a singular solution of type IIA theory, and should therefore be interpreted as a fundamental membrane in the latter theory. This finding pointed to a gap in the formulation of string theory that was subsequently filled by the discovery of the role of D-branes as the carriers of Ramond-Ramond charge in type II string theory. The roles of compactified eleven-dimensional membranes and fivebranes in five-dimensional string theory are also discussed.

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Low energy string theories contain a rich array of solutions corresponding to extended objects, the so-called p-branes (see [1] and references therein). It is now apparent that these objects play an important role in the non-perturbative physics of string theories [2]. To this end, a fundamental formulation of string theory that goes beyond the first quantized framework of the Polyakov path integral [3] is required. Progress into understanding the strong coupling dynamics of certain supersymmetric string theories has been made in [4], and which may provide new insights into the correct fundamental framework which must underly string theory.

This talk is divided into two parts. In the first, we follow [5] and present evidence that type IIA superstrings are only one component of a larger theory which also contains fundamental membranes, a finding that foreshadowed the subsequent discovery of the role of Dirichlet-branes (D-branes) as the carriers of Ramond-Ramond charge in type II string theory [6]. In the second part, we summarize the recent results of [7], in which a five-dimensional string is obtained from eleven-dimensional membrane and fivebrane solitons wrapped around two-cycles and four-cycles of a Calabi-Yau threefold.

According to string/string duality [8,9,4], the strong coupling physics of certain superstring theories may be reformulated as the weak coupling physics of "dual" string theories. One interesting example is the duality in six dimensions between heterotic strings compactified on T^4 and type IIA superstrings compactified on K3. In fact, the duality relies on the much stronger conjecture that these two strings are completely equivalent [9,4,10]. One further result which supports the equivalence is that the heterotic string can be identified as a soliton within the type IIA string theory, and conversely, the type IIA string can be identified as a soliton in heterotic string theory [11]. Thus under the duality transformation, the roles of the fundamental and solitonic strings are interchanged. This interchange is a stringy version of the role reversal between magnetic monopoles and electric charges arising in the strong/weak coupling duality of gauge field theories [12].

For a given p-brane solution of a string theory, the question arises as to how this solution behaves under the strong/weak coupling duality transformations discussed above. There are three distinct possibilities: (i) the p-brane could be a singular field configuration in both of the dual string theories, which would justify discarding it as unphysical, (ii) the p-brane could be nonsingular in both theories, in which case it would be treated as a soliton in both contexts, and finally (iii) the p-brane could be nonsingular in one theory but singular in the dual theory. In the latter case, since it appears as a soliton in one theory, one would not be able to omit it from the spectrum. However the fact that the

p-brane solution is singular in the dual theory suggests that it represents the external fields around a fundamental source¹ – i.e., the dual theory should contain fundamental p-branes!

The singularity structure of a solution is determined by examining the p-brane with a certain test-probe, *i.e.*, determining the behavior of a small test object as it approaches the core of the p-brane. The choice of the test-probe would depend on which fundamental theory underlies the original brane solution [13,1]. This amounts to measuring possible curvature singularities with the metric which couples to the world-volume of the fundamental objects in the theory, *i.e.*, the metric which appears in the sigma-model describing these fundamental objects. For example, in heterotic string theory, the natural test-probe to examine a p-brane solution would be a fundamental heterotic string. Applied to the case of the six-dimensional string/string duality, this means that the heterotic string appears singular in the heterotic string sigma-model metric, but is nonsingular in the type IIA superstring metric [11].

Consider heterotic string theory compactified on a torus down to six dimensions. For a generic point in the moduli space, the low energy effective theory is N=2 supergravity coupled to twenty abelian vector multiplets. Thus the bosonic fields include the metric, the dilaton, the antisymmetric Kalb-Ramond field, 24 abelian gauge fields, and 80 scalar moduli fields. In six dimensions the three-form field strength of the Kalb-Ramond two-form couples naturally as the "electric" or "magnetic" field around a one-brane, or string. In fact these correspond to the two string solutions discussed above, *i.e.*, the fundamental heterotic string with the electric Kalb-Ramond charge, and its dual solitonic string, with the magnetic three-form charge. Point-like or zero-brane solutions also appear, with conventional electric charges from the U(1) two-form field strengths. In particular, singular point-like objects arise as the extremal limits of electrically charged black holes. In this case, the dual objects are two-branes, or membranes, with magnetic U(1) charge. To complete the list, one could also consider three-branes which carry a "magnetic" charge from the periodic moduli scalars, and "minus-one"-branes or instantons carrying scalar electric charge. We restrict our attention, though, to a class of solitonic membranes.

It is consistent to truncate the low energy action to the form

$$S_{het} = \int d^6 x \sqrt{-G} e^{-2\Phi} \left(R + 4(\partial \Phi)^2 - \frac{1}{4} F^2 \right), \tag{1}$$

 $^{^{1}}$ In case (i), one could also consider the possibility that the p-brane is fundamental in both of the theories.

where F = dA is the field strength for one of the U(1) gauge fields, Φ is the six-dimensional dilaton and $G_{\mu\nu}$ is the heterotic string sigma-model metric. For this action, one finds the following solution which represents a magnetically charged membrane

$$ds^{2} = -dt^{2} + dx_{1}^{2} + dx_{2}^{2} + \left(1 + \frac{Q}{y}\right)^{2} \left(dy^{2} + y^{2}d\Omega_{2}^{2}\right),$$

$$e^{2\Phi} = 1 + \frac{Q}{y},$$

$$F_{\theta\varphi} = \sqrt{2}Q\sin\theta.$$
(2)

Here (y, θ, φ) are polar coordinates on the (x_3, x_4, x_5) subspace, and $d\Omega_2^2$ is the line element on the unit two-sphere².

While the metric in (2) may appear singular at the core of the membrane, this is a coordinate artifact. In fact, the solution develops an infinitely long throat with a constant radius as $y \to 0$, as is most easily recognized with the coordinate transformation $\rho/Q = \log(y/Q)$. Then the fields near the core become

$$ds^{2} \simeq -dt^{2} + dx_{1}^{2} + dx_{2}^{2} + d\rho^{2} + Q^{2} d\Omega_{2}^{2},$$

$$\Phi \simeq -\rho/2Q,$$

$$F_{\theta\varphi} = \sqrt{2}Q\sin\theta.$$
(3)

The above description of the throat geometry is made using the heterotic string sigmamodel metric, and hence this membrane is completely nonsingular for the heterotic string test-probes.

We are interested in considering this solution in the strong coupling regime in which the dual type IIA string theory is weakly coupled. Thus we seek a supersymmetric membrane saturating a BPS bound, for which the mass-charge relation is preserved against higher-order corrections in the strong coupling regime [15]. Therefore, we choose as our gauge field one of the four contained in the supergravity multiplet. This provides a supersymmetric embedding of (2) in the full six-dimensional N = 2 theory in which half of the spacetime supersymmetries are preserved [16]. In ten-dimensional heterotic string theory, this choice of gauge fields corresponds to setting $G_{i\mu} = B_{i\mu} = A_{\mu}$, where G and B denote

² This solution is simply the magnetically-charged extreme dilaton black hole from four dimensions [14], raised to six dimensions by adding the flat x_1, x_2 directions, which are tangent to the membrane.

the ten-dimensional metric and Kalb-Ramond field, respectively, with $\mu=0,1,2,3,4,5,$ a spacetime index and i=6,7,8 or 9 corresponding to one of the directions compactified on the four-torus. The ten-dimensional throat solution is then a constant radius three-sphere supported by the parallelizing torsion of the Kalb-Ramond field, a linear dilaton background in the ρ direction, and five flat spatial directions and a trivial time direction. This corresponds precisely to the throat limit of the ten-dimensional neutral fivebrane solution [17], and so reveals that our membrane is in fact a fivebrane "warped" around the toroidally compactified directions³. The throat solution is essentially unchanged, and one is guaranteed that no singularities develop at the membrane core [5]. Thus despite the appearance of α' corrections, we are assured that the membrane is a stable soliton of the heterotic string We also expect that the background Killing spinors are perturbatively corrected so that spacetime supersymmetry also survives the α' corrections [5].

We now transform the membrane soliton to the type IIA string theory via the duality mapping [4]

$$\Phi' = -\Phi,$$
 $G'_{\mu\nu} = e^{-2\Phi}G_{\mu\nu},$ $A'_{\mu} = A_{\mu}.$ (4)

Here the (un)primed fields are those arising in the type IIA (heterotic) string theory. In particular, $G'_{\mu\nu}$ is the metric which couples to the type IIA string sigma-model. The type IIA action is then given by

$$S_{IIA} = \int d^6x \sqrt{-G'} \left[e^{-2\Phi'} \left(R' + 4(\partial \Phi')^2 \right) - \frac{1}{4} F'^2 \right], \tag{5}$$

and the solution becomes

$$ds'^{2} = \left(1 + \frac{Q}{y}\right)^{-1} \left(-dt^{2} + dx_{1}^{2} + dx_{2}^{2}\right) + \left(1 + \frac{Q}{y}\right) \left(dy^{2} + y^{2} d\Omega_{2}^{2}\right),$$

$$e^{2\Phi'} = \left(1 + \frac{Q}{y}\right)^{-1},$$

$$F'_{\theta\omega} = \sqrt{2}Q\sin\theta.$$
(6)

In this frame the leading order solution becomes singular, requiring a source to support it at the core. First, the core, *i.e.*, y = 0, is a finite proper distance away, and the curvature

³ The solution is a "warped" as opposed to "wrapped" fivebrane [18,8]. The latter dimensionally reduces to an $a = \sqrt{3}$ black hole/H-monopole [19] in D = 4 as opposed to the a = 1 solution we started with in this paper. In our warped solution one of the compact directions is tied up in the three-sphere surrounding the fivebrane in a topologically nontrivial way.

diverges there, e.g., the Ricci scalar goes as $R \sim 1/(Qy)$. Thus from the point of view of type IIA string test probes, the membrane appears singular. Essentially with (4), we have made a singular conformal transformation of the original metric which implicitly adds an extra "point-at-infinity" closing off the end of the throat. To consistently solve the new equations of motion for (5), we must now include a source at this end-point, i.e., y = 0. Hence in the type IIA theory, the membrane must be interpreted as fundamental.

So the nonsingular supersymmetric solution in the heterotic string theory is singular in type IIA theory. Because of the nonsingular nature of the solution, it appears that these field configurations must be included in defining the heterotic string theory. From this and related results, three possible alternatives to describe the complete type IIA theory were suggested in [5]:

All-branes: an egalitarian theory of branes:— In this, the simplest alternative, the full type IIA theory is a theory which contains (at least) two distinct fundamental objects, strings and membranes. First quantization would be separately applied for each brane with its distinct world-volume action. A second step would be to incorporate interactions between the different branes in this first quantized framework. Presumably in this theory, the membranes would not contribute to the massless spectrum at a generic point in the (known) vacuum moduli space, since the latter spectrum is fully accounted for by type IIA strings. In this case, the membranes would play no role in the low energy physics, but would be important for a consistent definition of the theory at the level of massive modes and through nonperturbative effects [2]. Such an egalitarian description the type IIA theory was advocated in [20], where in fact on the basis of U-duality the democracy was extended to all p-branes appearing in the low energy theory.

Big-branes: a theory of only higher branes:— In this second scenario, the true type IIA theory would actually be a theory of only membranes (or some higher p-branes). The fundamental strings would then be "string-like" excitations of the membrane. In order for this alternative to be consistent, the membranes must also be able to act as sources for the Kalb-Ramond fields that are associated with the fundamental type IIA string. This requirement could be confirmed by examining the zero-mode structure of these solutions. Further, a much more stringent constraint is that consistently quantizing the fundamental membranes must reproduce precisely the same massless spectrum as the type IIA string in this K3 context. A higher brane description of the type IIA theory was advocated in [21] with the suggestion that the correct fundamental theory was an eleven-dimensional supermembrane theory.

Something else:— On this alternative, little was said in [5]. However we note that past efforts at quantizing higher p-branes have met with no success. Further even if a free first-quantized theory was constructed, the introduction of interactions for higher p-branes would remain a significant challenge. These technical obstructions lend favor to the opinion that only one-branes, or strings, should be treated as fundamental. The present analysis, which indicates that the type IIA theory must incorporate fundamental membranes, may then be an indication that the correct fundamental description of the theory is simply not one based on the first quantization of extended objects.

Interestingly, soon after [5] appeared, Polchinski [6] came up with the proposal that Dirichlet-branes (*D*-branes), extended objects defined by mixed Dirichlet-Neumann boundary conditions, are the carriers of electric and magnetic Ramond-Ramond charge. Once *D*-branes are added as Ramond-Ramond sources to type *II* string theory, the questions raised above are effectively answered. In particular, as a carrier of Ramond-Ramond charge, the membrane constructed above has the interpretation of a *D*-brane, and its mass per unit area has the correct dependence on the string coupling constant [5]. As *D*-branes will appear throughout these proceedings, we will not discuss them here in any detail. Suffice to say that their discovery fits in nicely with the simple physical picture described above in showing that, if string/string duality is to be taken seriously, the present formulation of string theory as a theory of only strings is insufficient, and that fundamental membranes are required to couple in some manner in order to complete the picture.

Recent activity has also focused on the conjecture of the existence of an underlying eleven-dimensional theory (the so-called M-theory [4,22–26]), whose low-energy limit is eleven-dimensional supergravity. M-theory also clearly fits in with the above discussion, and its eventual construction should lead to the establishment of the various string/string dualities [8,27,9,4,28,26]. In this framework, the five seemingly distinct string theories arise as weak coupling limits of the various compactifications of the eleven-dimensional M-theory, in which the membrane and fivebrane that naturally arise are either wrapped around or reduced on the compactified directions. In the rest of this talk, we summarize the recent results in [7], in which evidence is presented for a five-dimensional duality between M-theory compactified on a Calabi-Yau threefold and heterotic string theory compactified on $K3 \times S^1$.

In [29], the conjecture was made that the effective theory of heterotic string theory compactified on $K3 \times S^1$ is dual to eleven-dimensional supergravity compactified on a Calabi-Yau threefold. This theory is also equivalent to type IIA string theory compactified

on the same Calabi-Yau threefold, in an appropriate large volume limit. Point-like (electric) states are obtained in D=5 by wrapping the membrane from M-theory around two-cycles in the Calabi-Yau space. Denote two-cycles and four-cycles respectively by $C^{2\Lambda}$ and $C_{4\Lambda}$, where $\Lambda=1,...,h_{(1,1)}$. The charges of these states are obtained from the charge of the membrane by

$$e_{\Lambda} = \int_{C_{4\Lambda} \times S^3} G_7,\tag{7}$$

where $G_7 = \delta \mathcal{L}/\delta F_4$, where $F_4 = dA_3$ is the field strength of the three-form antisymmetric tensor field. String-like (magnetic) states in D = 5 arise by wrapping the fivebrane around four-cycles in the Calabi-Yau space. The charges of these states are then obtained from the charge of the fivebrane by

$$m^{\Lambda} = \int_{C^{2\Lambda} \times S^2} F_4. \tag{8}$$

Since the membrane and fivebrane are electric/magnetic duals in eleven dimensions, the above point-like and string-like states are dual to each other in the electric/magnetic sense and correspond to point-like and string-like soliton solutions [9]. Following the singularity structure criteria used to discuss the membrane above, one can show that in D = 5, each object is self-singular and mutually non-singular with its dual.

In a recent paper [26], heterotic string/string duality was examined from the point of view of M-theory, where it was argued that the $E_8 \times E_8$ heterotic string compactified on K3 with equal instanton numbers in the two E_8 's is self-dual, a result that can be seen by looking in two different ways at eleven-dimensional M-theory compactified on $K3 \times S^1/Z_2$. One weakly coupled heterotic string is obtained by wrapping the D=11membrane around S^1/Z_2 , while the dual heterotic string, also weakly coupled, is obtained by reducing the D=11 fivebrane on S^1/Z_2 and then wrapping around K3. Each of these two strings is strongly coupled from the point of view of the dual one. If we further compactify by reducing the first six-dimensional heterotic string on S^1 and wrapping the dual six-dimensional heterotic string on S^1 , we obtain on the one hand a string in five dimensions and on the other a dual, point-like object in five dimensions. We claim [7] that, starting with a K3 vacuum in which the gauge symmetry is completely Higgsed, this D=5 string can be identified with the M-theory fivebrane wrapped around a Calabi-Yau four-cycle, while the D=5 point-like object can be identified with the M-theory membrane wrapped around a Calabi-Yau two-cycle for the specific Calabi-Yau manifold $X_{24}(1,1,2,8,12)$ with $h_{(1,1)}=3$ and $h_{(2,1)}=243$ [30]. In five dimensions, this model

contains $n_V = h_{(1,1)} - 1 = 2$ vector multiplets (not counting the graviphoton) and $n_H = h_{(2,1)} + 1 = 244$ hypermultiplets.⁴

It is straightforward to match the perturbative and non-perturbative BPS states arising from the ten-dimensional compactification with the states displayed in the previous section and arising from the eleven-dimensional compactification. From the ten-dimensional point of view, the heterotic string compactified on $K3 \times S^1$ has the perturbative fundamental string state with charge

$$m_0 = \int_{K3 \times S^1 \times S^2} H_7,\tag{9}$$

where $H_7 = e^{-\phi} * H_3$, H_3 is the field strength of the two-form antisymmetric tensor field and ϕ is the ten-dimensional dilaton. This state has mass per unit length $M_0 = m_0 g_5^2$. Here the string is not wrapped around the S^1 . The corresponding classical solution is given by the fundamental string of [15]. This mass formula, which follows from central charge/supergravity considerations [29], can also be obtained by computing the ADM mass of the fundamental string solution. This state is associated with the $b_{\mu\nu}$ field and is dual to a vector in D = 5. The string theory also possesses a perturbative electrically charged point-like H-monopole state (dual to the magnetically charged H-monopole state of [19]) with charge

$$e_1 = \int_{K3 \times S^3} H_7 \tag{10}$$

and with mass $M_1 = e_1 R g_5$, where R is the radius of the S^1 and g_5 is the five-dimensional string coupling constant. In this case, the string is wrapped around the S^1 . Again one obtains the same mass from either the central charge or the ADM mass of the solitonic solution. This state is associated with the $b_{\mu 6}$ field. The T-dual electrically charged point-like Kaluza-Klein state with charge e_2 and associated with the $g_{\mu 6}$ field has mass $M_2 = e_2 g_5 / R$. In this case, the corresponding electrically charged solution is given by the extremal Kaluza-Klein black hole solution of heterotic string theory [31]. The fundamental string state can be identified with one of the three states arising from the M-theory fivebrane, while the H-monopole and Kaluza-Klein states can be identified with two of the three states arising from the M-theory membrane.

⁴ Here we do not consider the hypermultiplet sector of M-theory where the low-energy effective action in D = 5 does receive membrane and fivebrane instanton corrections [2].

The dual case is similar: the heterotic fivebrane wrapped around $K3 \times S^1$ has the non-perturbative (from the string point of view) point-like state with charge

$$e_0 = \int_{S^3} H_3 \tag{11}$$

and mass $M'_0 = e_0/g_5^2$ [4,29]. Here the classical solution is simply the heterotic fivebrane of [17] wrapped around $K3 \times S^1$, and which is dual to the fundamental heterotic string. One also gets from the heterotic fivebrane a non-perturbative magnetically charged string-like H-monopole state with charge

$$m_1 = \int_{S^1 \times S^2} H_3 \tag{12}$$

and mass per unit length $M'_1 = m_1 R/g_5$, where in this case the fivebrane is wrapped around the K3 but reduced on the S^1 . The solution in this case is the usual magnetically charged H-monopole, which in D=5 is a string [19]. The T-dual magnetically charged string-like Kaluza-Klein state with charge m_2 has mass per unit length $M'_2 = m_2/g_5 R$. The point-like state can be identified with one of the three states shown in the previous section arising from the M-theory membrane, while the string-like H-monopole and Kaluza-Klein states can be identified with two of the three states shown in the previous section arising from the M-theory fivebrane.

Note that each of the three pairs of electric/magnetic dual states obey Dirac quantization conditions. Note also that neither the membrane nor the fivebrane from M-theory is in itself sufficient to reproduce the perturbative spectrum of either the five-dimensional string or the dual five-dimensional point-like object. This becomes clear when one realizes that, from the M-theory side, the membrane wrapped around a two-cycle yields only point-like states, while the fivebrane wrapped around a four-cycle yields only string-like states. On the other hand, from the heterotic compactification, both the string and point-like theories in D=5 contain both string and point-like objects in their perturbative spectra. In particular, it follows that the D=5 spectrum of Calabi-Yau string solitons yields the fundamental string states on the heterotic side as well as the non-perturbative heterotic string states obtained by wrapping the heterotic fivebrane on K3.

One-loop calculations providing further evidence for this duality were shown in [7]. It was also found that, from anomaly considerations, a five-dimensional string action arises which is chiral on the worldsheet. This is especially interesting, since it implies that M-theory calculations may be carried out in the more familiar setting of string theory. Further reduction to D=4 yields the standard dual N=2 supersymmetric theories, but one may

hope to obtain dual N=1 chiral theories following [26] by considering two different limits of M-theory compactified on $CY \times S^1/Z2$.

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